Physics 211C: Solid State Physics

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Lecture 20

Topic: Quantum-Hall continued

Fractional charge, statistics (anyons) & g.s.d

in abelian FQH states

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disorder is imp. yet we'll ignore it. ⇒ Imp. for IQHE & also for FQHE

Low en. theory

irrelevant at low energies

$$\mathcal{L} = \frac{k}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + eA_\mu \underbrace{\frac{\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda}{2\pi}}_{\text{electric current}} + f_{\mu\nu}f^{\mu\nu}$$

$(\nabla\times\vec{a} = \vec{j})$ & by const. $\nabla\cdot\vec{J} = 0$

$$e^{iS} \qquad S = \int d^2x\, d\tau\, \mathcal{L}$$

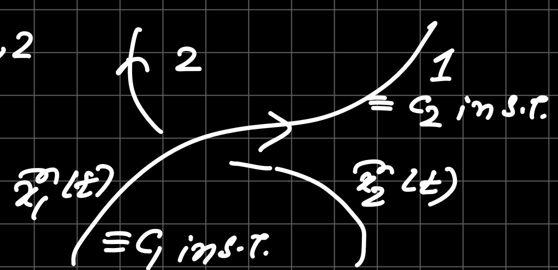
$$\tau = it$$

$$\mathcal{L} = \frac{k}{4\pi}\epsilon_{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \underbrace{j_1^\mu a_\mu + j_2^\mu a_\mu}_{2\text{ QPs}}$$

$$j^0_\alpha = \delta(\vec{x} - \vec{x}_\alpha(t)) \qquad \alpha = 1,2$$

$$j^i_\alpha = \dot{x}^i_\alpha\, \delta(\vec{x} - \vec{x}_\alpha(t))$$

$$i = x, y$$



$$Z = \int \mathcal{D}a\, e^{iS} = \exp\left(-\frac{2\pi i}{k} \times \text{linking num}(C_1, C_2)\right)$$

Exchange statistics of q.p. that carry gauge charge 1 of

$$\theta = \frac{\pi}{k}$$

Calculat^n

$$Z = \int \mathcal{D}a\; e^{i\int \frac{k}{4\pi}\epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + i\int j_1^\mu a_\mu + i\int j_2^\mu a_\mu}$$

$$Z = e^{i\int d^2x\, d\tau\; j_1^\mu \frac{1}{\frac{k}{2\pi}\partial_\lambda \epsilon^{\mu\nu\lambda}} j_2^\mu}$$

call $\dfrac{1}{\frac{k}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\lambda} j_2^\nu = f_2^\mu$

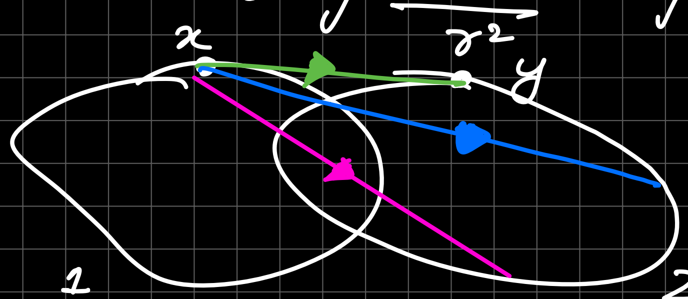
$$\Rightarrow \frac{k}{2\pi}\; \underbrace{\epsilon^{\mu\nu\lambda}\partial_\lambda f_2^\mu = j_2^\mu}_{\nabla\times\vec{B} = \vec{J}}$$

so now find $\vec{B}$ from $\vec{J}$ using Biot-Savart law.

$$Z = e^{i\, d^2x\, d\tau\; j_1^\mu \underbrace{f_2^\mu}_{\sim B}}$$

$$Z = e^{-\frac{2i}{2k}\int d^3x\, d^3y\, \frac{(x-y)^\nu}{|x-y|^3} j_1^\mu(x)\, \epsilon_{\mu\nu\lambda}\, j_2^\lambda(y)}$$

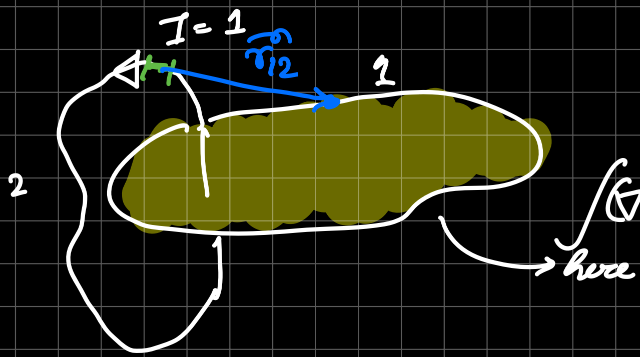
$$B \sim \int \frac{I\, d\vec{l} \times \hat{r}}{r^2} \sim \int \frac{I\, d\vec{l} \times \vec{r}}{r^3}$$



integrat^n is over all such pairs of points.

Claim:- above # is a topological number.

$$Lk(C_1,C_2) = \frac{1}{4\pi}\int_{C_1}\int_{C_2} \frac{(\vec{r}_{12}\times d\vec{r}_2)\cdot d\vec{r}_1}{r_{12}^{3}}$$



$$\nabla\times\vec{B} = \vec{J}$$

$$\int(\nabla\times\vec{B})\cdot d\vec{s} = \oint \vec{J}\cdot d\vec{s}$$

$$= I\times Lk(C_1,C_2)$$

$$\Rightarrow \oint_{C_1}\vec{B}\cdot d\vec{l} = Lk(C_1,C_2)$$

$$\Rightarrow \oint \frac{(\vec{r}_{12}\times d\vec{r}_2)}{r_{12}^{3}}\cdot d\vec{r}_1 = Lk(C_1,C_2)$$

Biot-Savart law is topological & so is CS.

Ref:- Geometry of Physics ⇒ Fraenkel

$$\boxed{Z = e^{-\frac{2\pi i}{k} Lk(C_1,C_2)}}$$ → $k>1$, fractional statistics.

Fractional Charge

* Why call a Lagrangian a state?

Ref:- Wen's book